W projection

A new algorithm for wide field imaging with radio synthesis arrays

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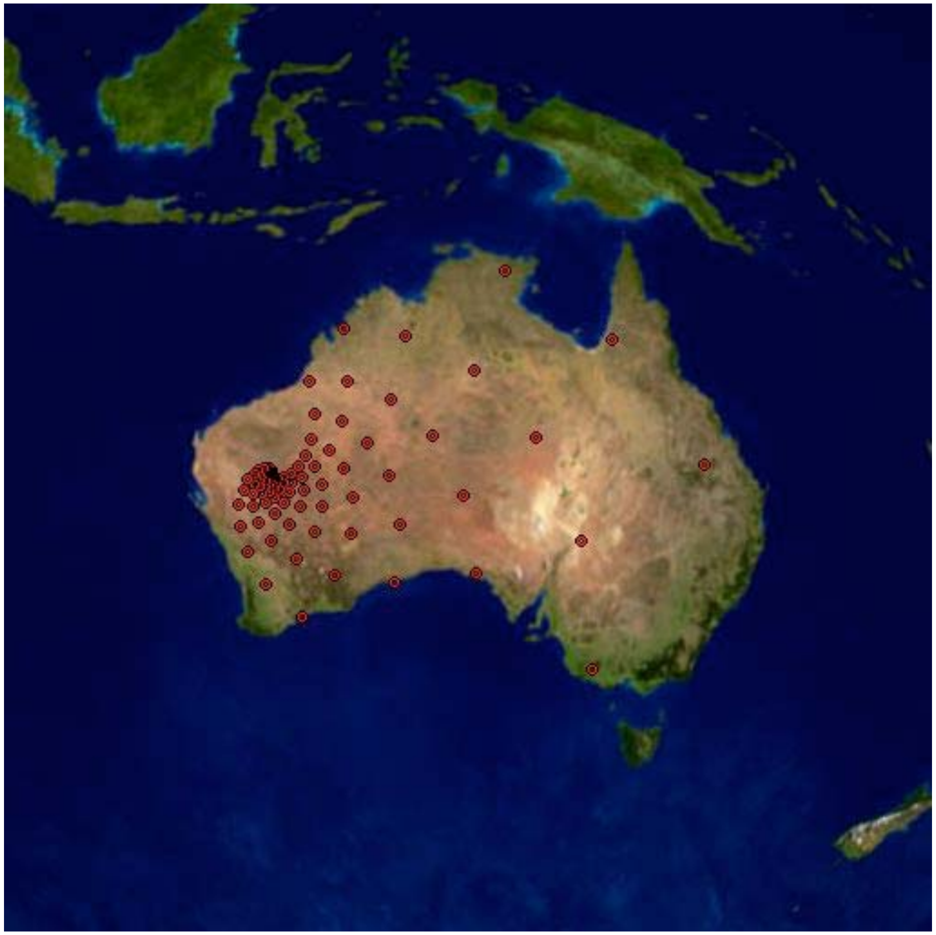
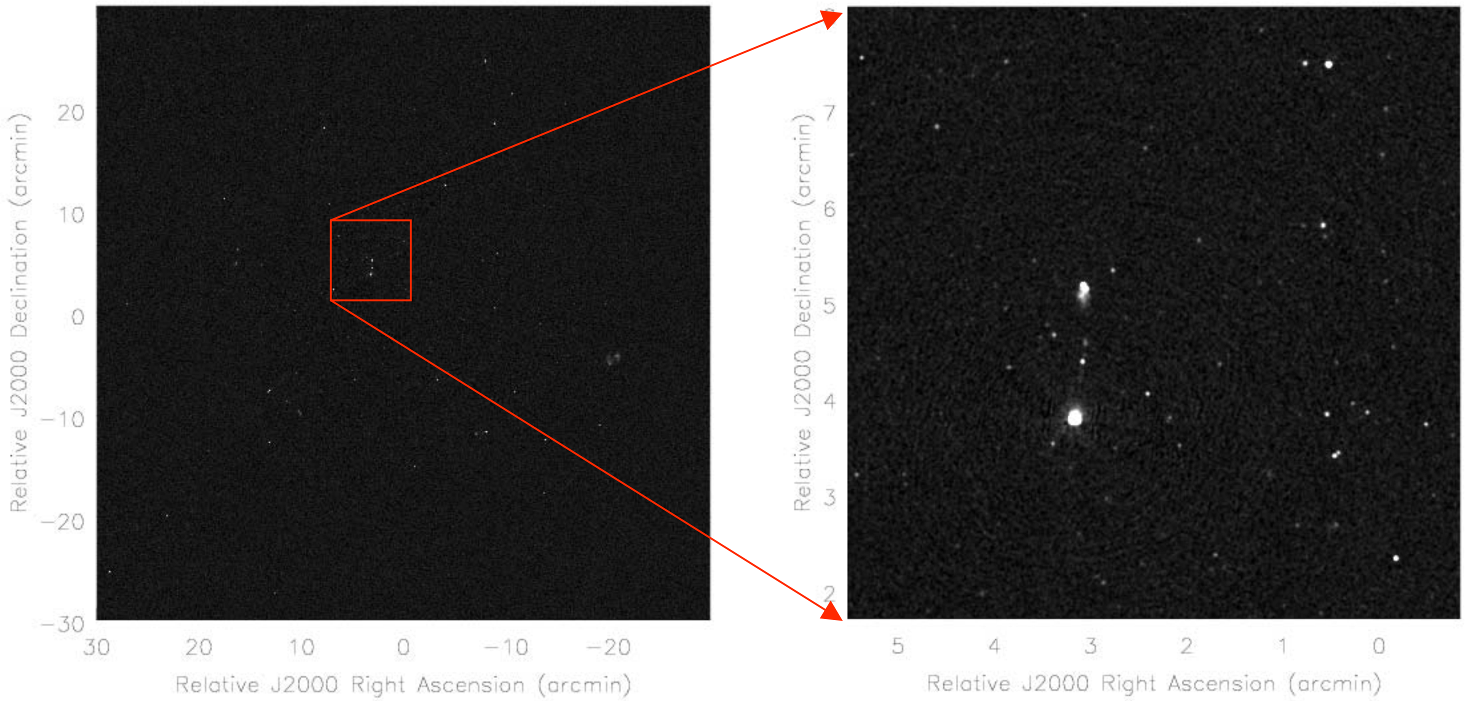
Square Kilometre Array

- Described by Shep Doeleman this morning
- Baselines up to 3000km
- Few hundred stations on baselines form 150km to 3000km
- Frequency 0.1 - 25GHz
- Many challenges in calibration and imaging

Possible configuration
in Australia



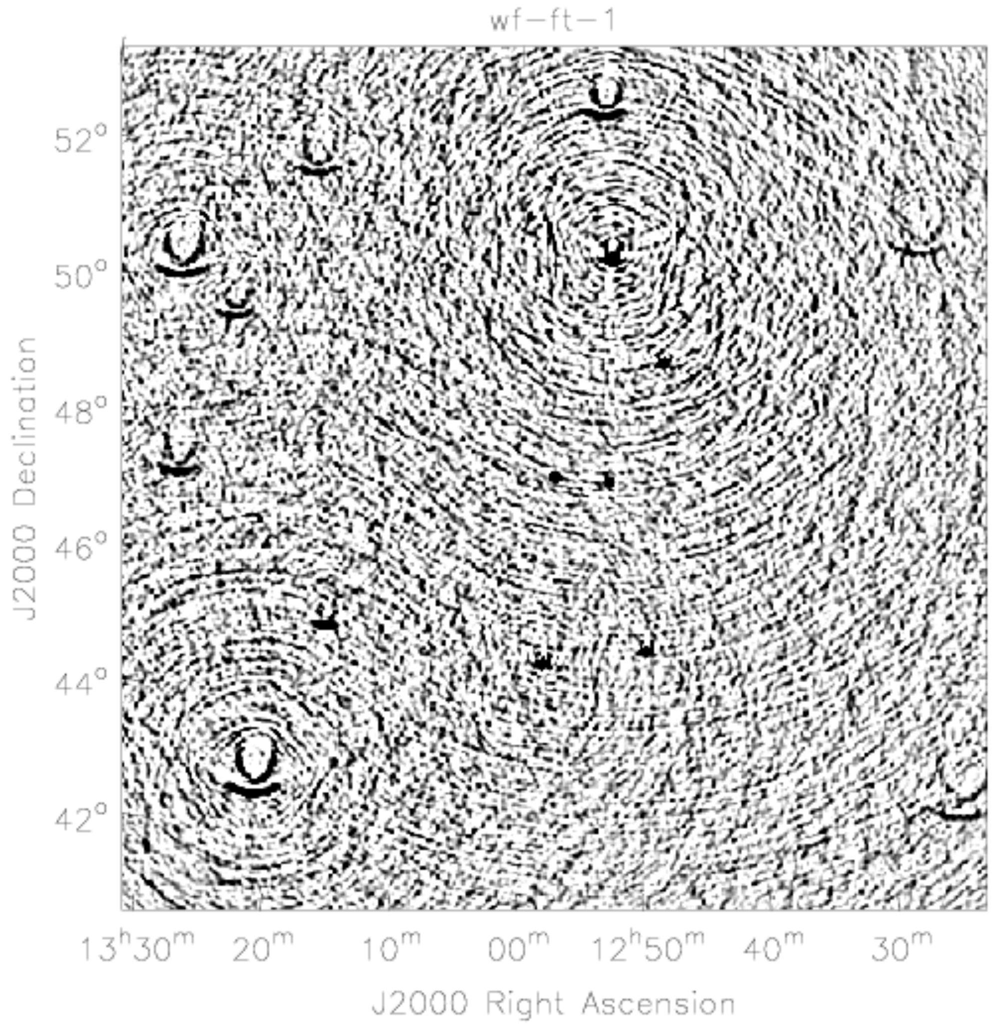
3μJy/beam 1.5arcsec 1.4GHz VLA image



- Square Kilometre Array will be ~ 100 times deeper
- Confusion limits probably require 0.1arcsec resolution
- To reach sensitivity limit, must image accurately all emission over 1 degree FOV
- Image sizes could be up to 80,000 by 80,000 pixels

A problem....

- Point sources away from the phase center of a radio synthesis image are distorted
- Bad for long baselines, large field of view, and long wavelengths
- Algorithms exist and work
  - But slowly....
- Will be a substantial problem for SKA

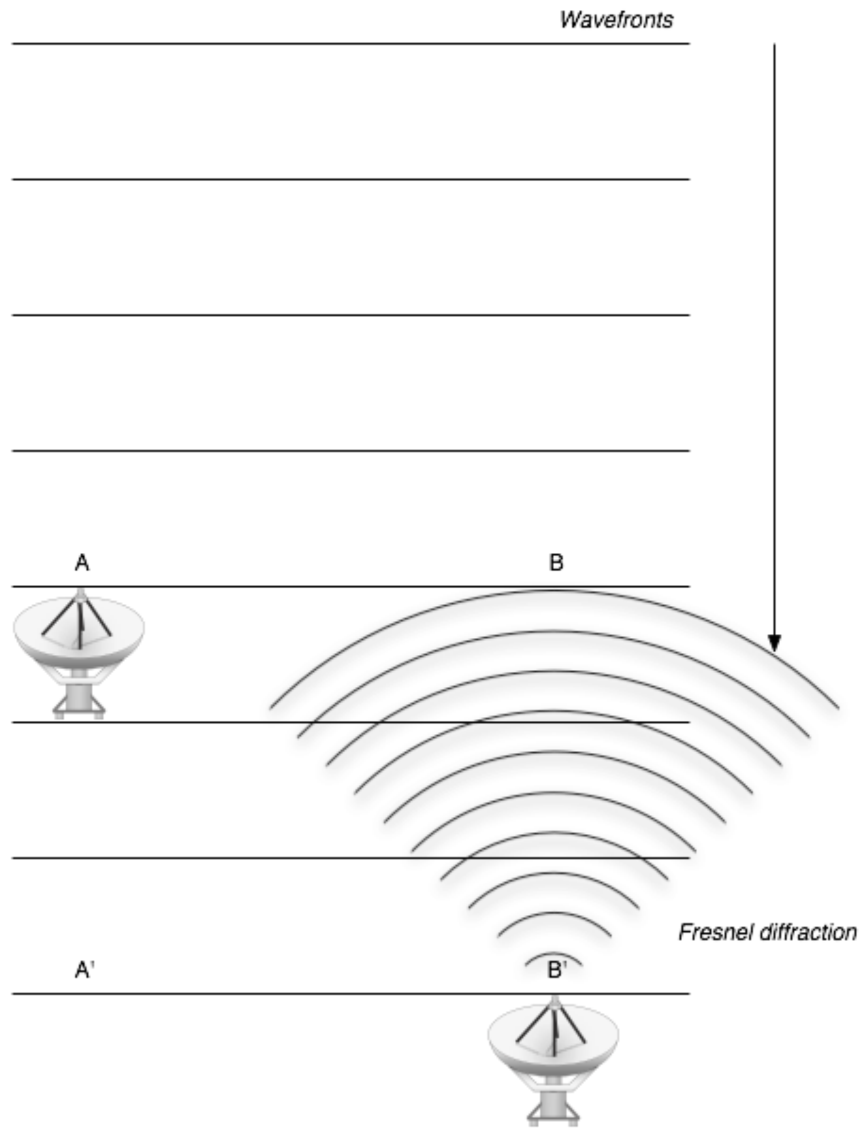


A simple piece of optics...

If we had measured on plane AB then the visibility would be the 2D Fourier transform of the sky brightness

Since we measured on AB', we have to propagate back to plane AB, requiring the use of Fresnel diffraction theory since the antennas are in each others near field

$$\tilde{G}(u,v,w) \approx e^{-j\pi w(u^2+v^2)}$$



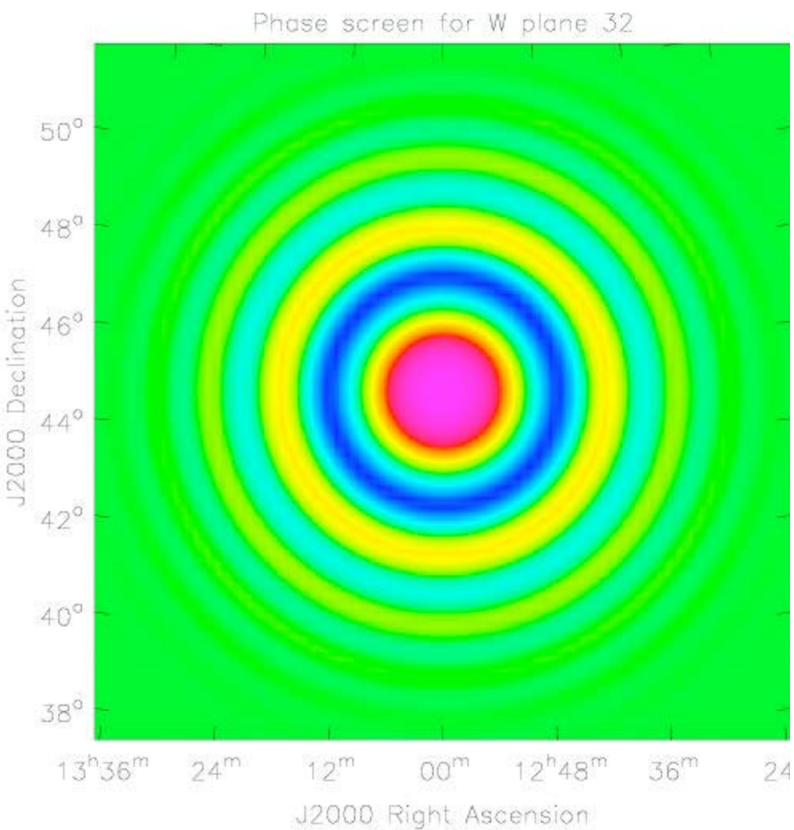
The convolution function

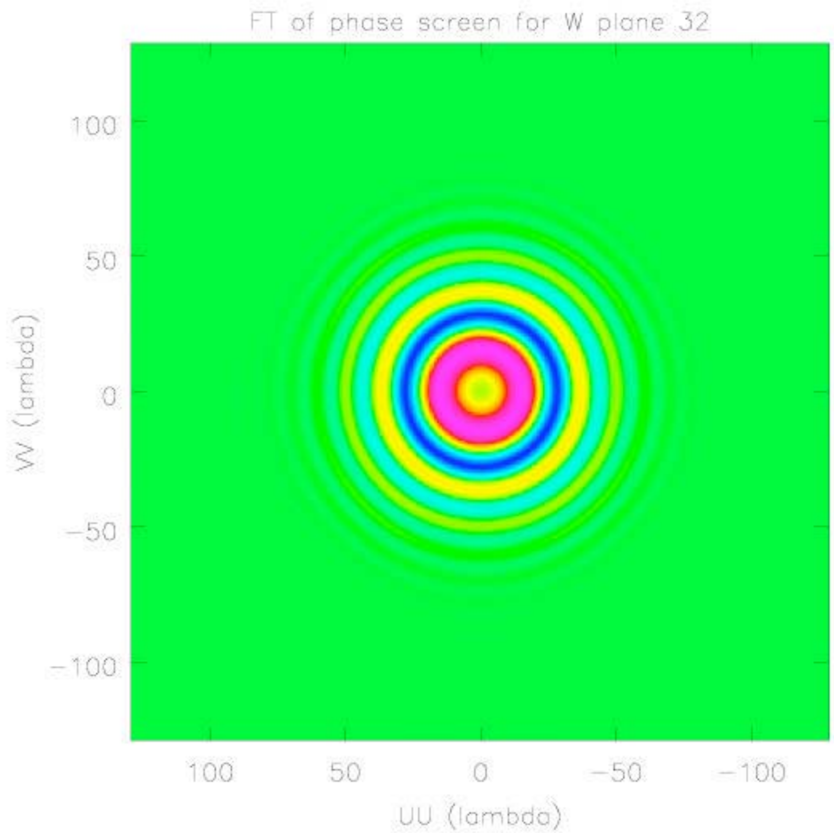
Image plane phase screen

$$e^{j2\pi w\left(\sqrt{1-l^2-m^2}-1\right)}$$

Fourier plane convolution function

$$\approx e^{-j\pi w(u^2+v^2)}$$





From narrow field to wide field

Standard narrow field measurement equation

$$V(u,v) = \int I(l,m) e^{j2\pi(ul+vm)} dl dm$$

Fresnel diffraction

$$V(u,v,w) = \tilde{G}(u,v,w) \otimes V(u,v)$$

Standard wide field measurement equation derived using Van Cittert-Zernike theorem

$$V(u,v,w) = \int I(l,m) e^{j2\pi\left(ul+vm-w\left(l^2+m^2\right)/2\right)} dl dm$$

$$\approx \int \frac{I(l,m)}{\sqrt{1-l^2-m^2}} e^{j2\pi\left(ul+vm+w\left(\sqrt{1-l^2-m^2}-1\right)\right)} dl dm$$

Wide field imaging = narrow field imaging + convolution

Fresnel scale $\sqrt{\lambda B}$

- Fresnel scale = size of region of influence
- If Fresnel scale > antenna diameter, measurements must be distorted

	Wavelength			
Baseline length	**4**	**1**	**0.21**	**0.06**
1000	63	32	14	8
3500	118	59	27	14
10000	200	100	46	24
35000	374	187	86	46
100000	632	316	145	77
350000	1183	592	271	145

Roughly the size of convolution function in pixels

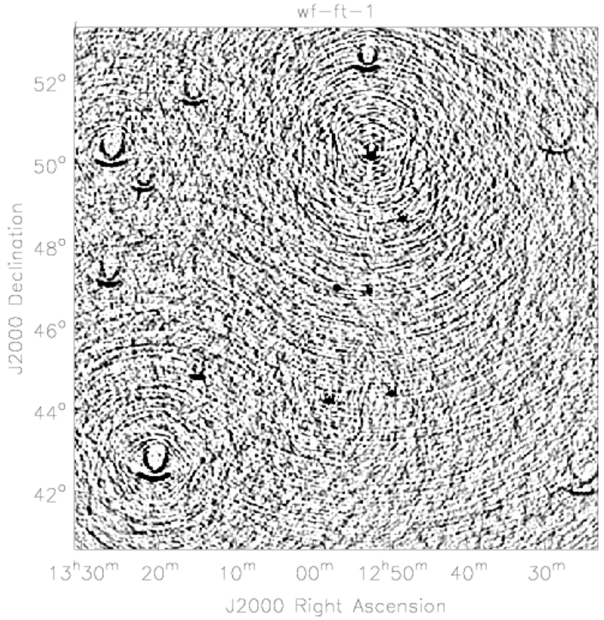
The W projection algorithm

- Calculate gridding kernel for range of values of $\sqrt{w}$
  - Fourier transform phase screens multiplied by spheroidal function (needed to control aliasing)
- Image to Fourier
  - Taper image by spheroidal function
  - Fourier transform
  - Estimate sampled visibilities by convolving gridded values with w dependent kernel
- Fourier to Image
  - Convolve sampled visibilities onto grid using w dependent kernel
  - Inverse Fourier transform
  - Correct for spheroidal function
- Deconvolution
  - Deconvolve in minor cycles using PSF for image center
  - Reconcile to visibility data in major cycles

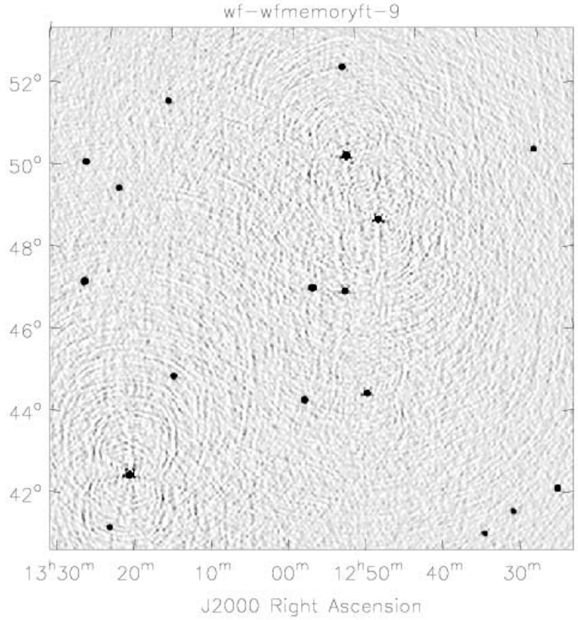
A synthetic example

- Simulation of ~ typical 74MHz field
  - Sources from WENSS
  - Long integration with VLA

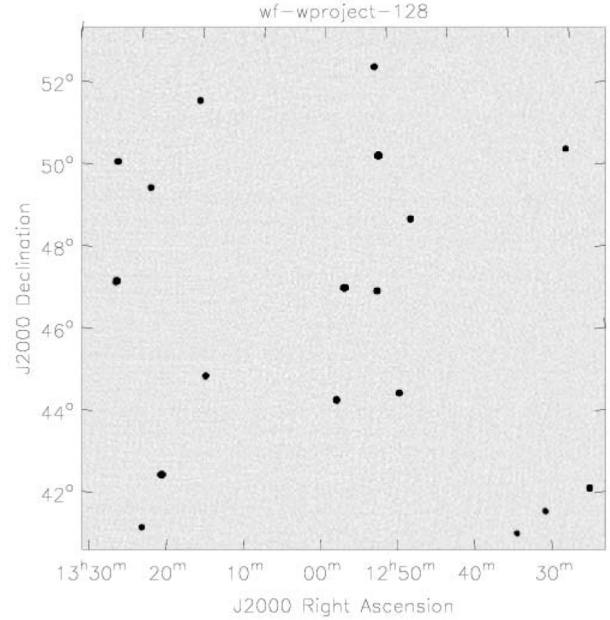
Fourier transform



UVW space facets



W projection



Is this fast enough?

$$C_{SKA} \quad \$3.5\mathrm{M}\left(\frac{0.1}{\eta}\right)\left(\frac{f}{0.5}\right)^{2}\left(\frac{B}{5km}\right)^{3}\left(\frac{D}{12.5m}\right)^{-8}\left(\frac{\lambda}{0.2m}\right)\left(\frac{\Delta\nu}{500MHz}\right)2^{\frac{2(2010-t)}{3}}$$

- Antenna diameter scaling is horrific!
  - Doubling antenna size saves factor of 256 in computing
- Baseline dependency is tough
- Easy to find hardware costs > SKA cost
- Multi-fielding not included
- Error ~ factor of 3 in each direction
- For 350km baselines with 25m antennas
  - *\$120M in 2015*
  - *Without w projection, would be ~ \$1B*

Escape routes

- Invent a new algorithm for wide-field imaging
  - Algorithm must be designed for hardware resources
  - W projection possible with ~ GB memory
  - Optimum approach for multi-processors likely to be quite different
- Correlator FOV shaping
  - Note by Lonsdale, Doeleman, and Oberoi (17 July 2004)
  - Does not interact well with calibration?
- Avoid small antennas
  - Very large antennas not good for imaging?
  - Lower risk by developing design for cheap 20m antenna
- Only do hard cases infrequently
  - Reinvest in computing periodically
- Special purpose hardware?
  - FPGA?
- Use stations on long baselines

Summary

- The non-coplanar baselines effect is caused by Differential Fresnel diffraction
- W projection corrects the non-coplanar baselines effect by convolving with Fresnel diffraction kernel in uvw space before Fourier transform
- W projection is an order of magnitude faster than facet based methods
- Non coplanar baselines effect is still a significant obstacle for SKA - stay tuned…

References, *etc.*

- See Poster P2.1.2 *"Mosaicing with interferometric radio telescopes: efficient algorithm for imaging and image plane corrections",* Sanjay Bhatnagar, Kumar Golap, and TC
- EVLA memos 67 and 77
- My home page:

  [*http://www.nrao.edu/~tcornwel*](http://www.nrao.edu/~tcornwel)